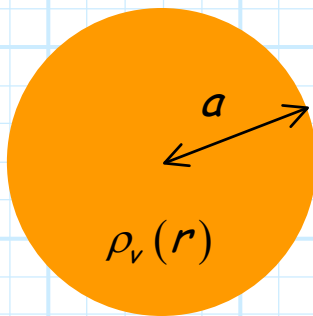


Example: Using Gauss's Law to Determine the Electric Field

Consider a "cloud" of charge with radius a and centered at the origin, described by volume charge density:



$$\rho_v(\vec{r}) = \begin{cases} \frac{1}{r} & r < a \\ 0 & r > a \end{cases}$$

Q: What electric field $\mathbf{E}(\vec{r})$ is produced by this charge?

A: We could use Coloumb's Law to solve this, but note that this is a **spherically symmetric** charge density! As a result, we can find the electric field much easier using **Gauss's Law**.

Recall that **spherically symmetric** charge densities produce an electric field:

$$\begin{aligned} \mathbf{E}(\vec{r}) &= \frac{Q_{enc}}{4\pi\epsilon_0 r^2} \hat{a}_r \\ &= \frac{\hat{a}_r}{\epsilon_0 r^2} \int_0^r \rho_v(r') r'^2 dr' \end{aligned}$$

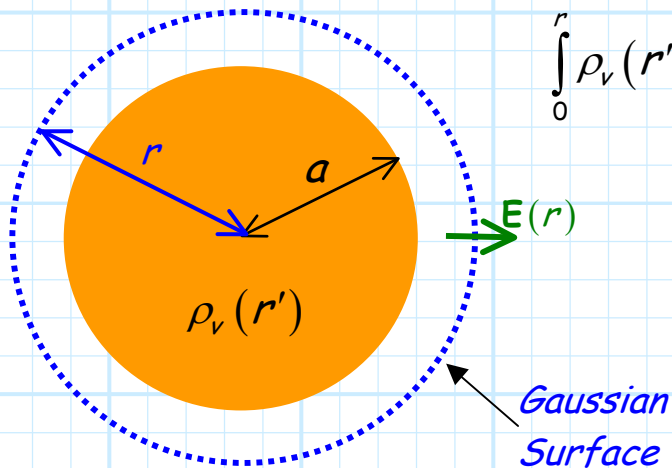
Evaluating the **integral**, we need to consider **two cases**: one where r (i.e., the radius of the **Gaussian surface**) is **less than** cloud radius a (for evaluating the field **within** the charge cloud), and the second where r is **greater than** cloud radius a (for evaluating the field **outside** the charge cloud).

For $r < a$:

The diagram shows a solid orange circle representing a charge cloud of radius a . A dashed blue circle inside it represents a Gaussian surface of radius r . A green arrow labeled $\mathbf{E}(r)$ points radially outward from the center. A label $\rho_v(r')$ is placed inside the cloud. A blue arrow points to the dashed circle with the label "Gaussian Surface".

$$\begin{aligned} \int_0^r \rho_v(r') r'^2 dr' &= \int_0^r \left(\frac{1}{r'} \right) r'^2 dr' \\ &= \int_0^r r' dr' \\ &= \left| \frac{r'^2}{2} \right|_0^r \\ &= \frac{r^2}{2} \end{aligned}$$

And if $r > a$:



$$\begin{aligned} \int_0^r \rho_v(r') r'^2 dr' &= \int_0^a \left(\frac{1}{r'} \right) r'^2 dr' + \int_a^r 0 r'^2 dr' \\ &= \int_0^a r' dr' + 0 \\ &= \left| \frac{r'^2}{2} \right|_0^a \\ &= \frac{a^2}{2} \end{aligned}$$

Therefore, the **electric field** produced by this charge is:

$$\begin{aligned} \mathbf{E}(\vec{r}) &= \frac{\hat{a}_r}{\epsilon_0 r^2} \int_0^r \rho_v(r') r'^2 dr' \\ &= \begin{cases} \frac{\hat{a}_r}{\epsilon_0 r^2} \left(\frac{r^2}{2} \right) & r < a \\ \frac{\hat{a}_r}{\epsilon_0 r^2} \left(\frac{a^2}{2} \right) & r > a \end{cases} \\ &= \begin{cases} \frac{\hat{a}_r}{2\epsilon_0} & r < a \\ \frac{\hat{a}_r}{2\epsilon_0} \left(\frac{a^2}{r^2} \right) & r > a \end{cases} \end{aligned}$$

Note the resulting electric field behaves **as expected**. The field points in the direction \hat{a}_r (i.e., points away from the origin). It is likewise independent of θ or ϕ (i.e., **spherically symmetric**).

Note also that the magnitude of the field outside of the cloud **diminishes** as $1/r^2$. This **makes sense!** Do you see why?